CYCLIC OPERATION OF UNDERGROUND GAS STORE

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Within the framework of the Buckley–Leverette model, an accurate solution is given for the problem of the combined filtration of two incompressible fluids in the course of successive pumping of one of them alternatively into and out of a porous collector.

The normal operation of underground gas stores is cyclic in character [1]. Each cycle consists of pumping a definite volume of gas into the bed and then removing it in an amount determined both by the requirement for the gas and by the filtrational characteristics of the gas-saturation collector. The basic problem of cyclic operation of an underground gas store is to elucidate how the store dimensions and the volume ratio of the gas pumped into and out of the bed change from cycle to cycle and how the volume of the so-called "trapped" gas, i.e., that which is inaccessible for removal, grows. In particular, the important question is whether the gas store works in steady conditions, i.e., conditions in which the dimensions of the gas store do not increase and the volume of gas pumped out is practically equal to the volume pumped in.

In [2, 3], attempts were made to obtain answers to these questions by means of specific graphoanalytic constructions, but no general solution was obtained. The investigation of the cyclic use of an underground gas store undertaken here is based on the Buckley–Leverette model of two-phase filtration. Accurate solution of the problem is performed under the assumption that a definite volume of gas is pumped into the bed each time, and pumping out continues until the water front approaches the borehole.

Consider the differential equation for the distribution of the saturation $\sigma(x, t)$ of one of the cofiltering fluids [4, 5]

$$m\,\frac{\partial\sigma}{\partial t}+\frac{w}{x^{\nu-1}}\,\frac{\partial f(\sigma)}{\partial x}=0, \tag{1}$$

where $\nu = 1, 2$, respectively, for linear and radial flow; $f(\sigma) = k_1(\sigma)/[k_1(\sigma) + \mu_0 k_2(\sigma)]$ is the Buckley–Leverette function. A characteristic feature of this function is that its curve has two parts, one convex and one concave, separated by a point of inflection σ_I (Fig. 1).

Suppose that the same volume of gas V is pumped into the bed in each cycle. Assume that w is constant; then the injection of the gas will always extend over the same time T. Introducing dimensionless variables according to the formulas $\xi = mx^\nu/\nu wT$ and $\tau = t/T$, Eq. (1) may also be written in dimensionless form

$$\frac{\partial\sigma}{\partial\tau}+f'(\sigma)\frac{\partial\sigma}{\partial\xi}=0. \tag{2}$$

It may now be assumed that the dimensionless injection time is unity. Note that, if the process of displacement of water by gas is described by the function $f(\sigma)$, the inverse process of gas displacement by water is described by the function $f_1(\sigma) = 1 - f(1-\sigma)$, where $f'(\sigma) = f_1'(1-\sigma)$.

Equation (2) belongs to the class of hyperbolic quasilinear equations, and therefore has real characteristics. The equation of these characteristics and the conditions imposed upon them take the form

$$\frac{d\xi}{d\tau}=f'(\sigma),\quad \frac{d\sigma}{d\tau}=0, \tag{3}$$

and hence it follows that they are straight lines. Equation (2) may have discontinuous solutions, therefore at the lines of discontinuity it is necessary to impose relations between the limiting values of the saturation "before" and "after" the discontinuity and relations expressing the mass balance of each of the phases [6]. In the given case, these relations are the same, and take the form

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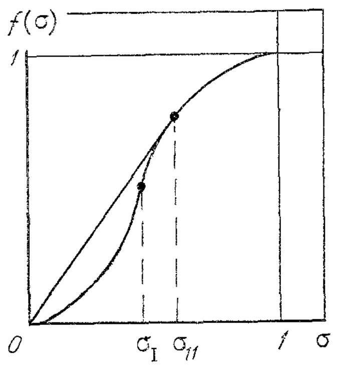


Fig. 1

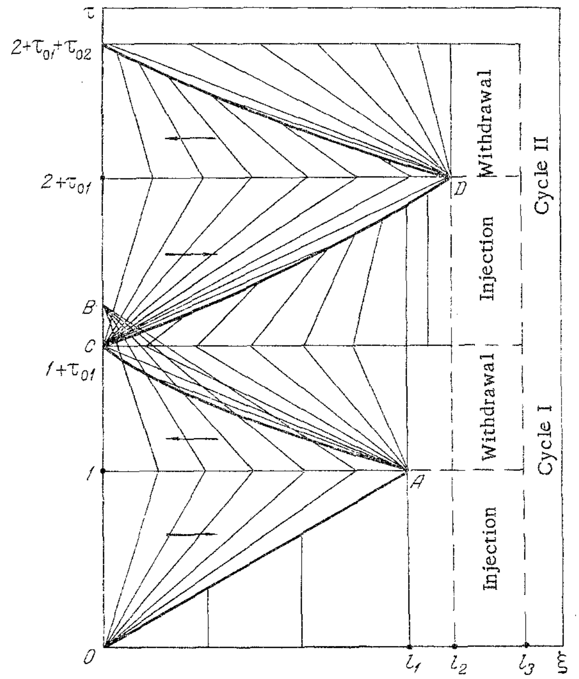


Fig. 2

Fig. 1. Buckley–Leverette function.

Fig. 2. Position of the characteristics and discontinuity lines on the (ξ, τ) plane; ξ and τ are the dimensionless coordinate and time.

$$D = \frac{d\xi}{d\tau} = \frac{f(\sigma^+) - f(\sigma^-)}{\sigma^+ - \sigma^-}, \tag{4}$$

where σ^+ and σ^- are the saturations before and after the discontinuity, respectively. In addition, such discontinuities must be stable [6].

The solution will now be constructed. The first injection of gas into the bed initially filled with water is described by Eq. (2) with the initial and boundary conditions $\sigma(\xi, 0) = 0$, $\sigma(0, \tau) = 1$. The solution is constructed in the region $\xi \geq 0$, $0 < \tau \leq 1$. This is the ordinary Buckley–Leverette self-similar problem; its solution consists of the discontinuity $\xi/\tau = f'(\sigma_{11})$, where σ_{11} is the frontal saturation, determined by the abscissa of the point of tangency of the straight line drawn through the point (0; 0) to the Buckley curve $f(\sigma)$ (Fig. 1) and the subsequent centered wave $\xi/\tau = f'(\sigma)$. Before the discontinuity the saturation σ is zero.

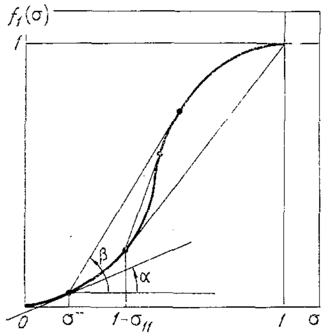
The picture of the characteristic and the position of the line of discontinuity are shown in the lower part of Fig. 2. The gas-store dimension l_1 after the first injection is determined by the relation $l_1 = f'(\sigma_{11})$, while the distribution of the saturation at the end of injection is found from the relation $\xi = f'(\sigma)$.

When the gas is pumped out, the direction of filtration of the phases is reversed, and therefore, for the sake of convenience, the coordinate origin may be shifted to the point reached by the displacement front at the end of injection. Then, the withdrawal of the gas is described by the equation given above, with the difference that the gas saturation is replaced by the water saturation, and the function $f(\sigma)$ by the function $f_1(\sigma)$. However, the problem of gas withdrawal is not self-similar. The distribution of the saturation formed in the bed at the end of injection $\sigma(1, \xi) = \sigma(\xi)$ characterizes the initial condition for the given equation

$$l_1 - \xi = f_1(\sigma), \quad 0 \leqslant \xi \leqslant l_1. \tag{5}$$

Here $\sigma \in (0; 1 - \sigma_{11})$ and, in addition, $\sigma(0, \tau) = 1$, $\tau \geq 1$.

The distribution of saturation in this stage of the process is also found by the method of characteristics. It may readily be shown that all the characteristics leaving points of the initial straight line converge at a single point C (Fig. 2). This is explained in that the initial distribution in Eq. (5) is obtained from the solution of the same equation, which is a centered wave. It is necessary to construct a stable discontinuous transition from these characteristics at the value ($\tau = 1$). It consists of the discontinuity (AC) traveling over the region



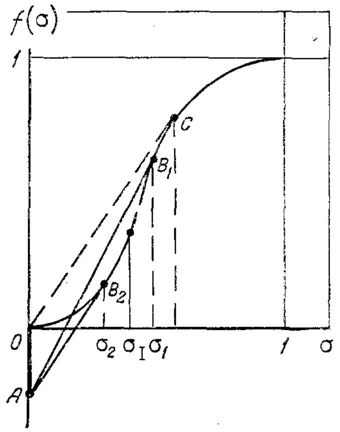


Fig. 3 Fig. 4

Fig. 3. Comparison of the velocity of the discontinuity and the perturbations: $\tan\alpha = f'(\sigma^-)$; $\tan\beta = D$, $D > f'(\sigma^-)$.

Fig. 4. Geometric interpretation of the results obtained.

of convergent characteristics and the subsequent centered wave adjacent to the boundary at which the saturation is constant: $\sigma = 1$. It is obvious that this discontinuity is stable [6]. First, it overtakes all the characteristics traveling in front of the discontinuity. In fact, the rate of such characteristics, according to Eq. (3), is determined by the values of the derivative $f_1'(\sigma^-)$ and the rate D of the discontinuity by Eq. (4). Since $\sigma \in (0; 1 - \sigma_{11})$, then $D > f_1(\sigma^-)$ (Fig. 3), i.e., the discontinuity intersects all the characteristics proceeding from the interval $(0; l_1)$. Hence, it also follows that the time of withdrawal τ_{01}, i.e., the time in which the saturation discontinuity reaches the borehole, is less than unity. Second, it may be shown that the discontinuity (AC) moves more slowly than the perturbations behind it, and the characteristics of the centered wave lying behind it overtakes the discontinuity, reducing its velocity. Thus, the stability condition of [6] is satisfied.

The second gas injection is described by Eq. (2) in the previous reference frame. This problem is also non-self-similar. If the value of the water saturation at the moment at which the water front (discontinuity) reaches the borehole is denoted by σ_{12}, the initial and boundary conditions for this equation will take the form

$$\frac{l_1 - \xi}{1 + \tau_{01}} = f'(\sigma),\ \sigma \in [0;\ 1 - \sigma_{12}];\ \sigma(0,\ \tau) = 1,\ \tau > 1 + \tau_{01}.$$

Thus, the problem is found to be analogous to the preceding one; specifically, in the interval $(0; l_1)$, there is an initial distribution of the saturation obtained from the solution of the problem at the stage of gas withdrawal in the preceding cycle, i.e., from the centered wave subsequent to the discontinuity (AC). A discontinuity (CD) begins to propagate from this boundary $\xi = 0$ (the borehole) against this background; its velocity here will be larger than that of the perturbations preceding it. Hence, it follows that the point $\xi = l_1$ is reached by the discontinuity more rapidly than characteristics starting from points of the interval $(0; l_1)$ converge at the point $(l_1;\ 1 + 2\tau_{01})$. Behind the discontinuity (CD), there moves a centered wave, rays of which overtake the discontinuity and slow it down (see Fig. 2). Since the injection time (unity) is larger than the withdrawal time τ_{01} the gas front does not stop at the point $\xi = l_1$, but will move further and, at $\tau = 2 + \tau_{01}$, reaches the point $\xi = l_2$ $(l_2 > l_1)$. In this case, the gas saturation preceding the discontinuity is zero in the interval $(l_1; l_2)$, and its motion is described by the ordinary differential equation in Eq. (4), with σ^+ taken from the expression for a centered wave and $\sigma^- = 0$.

At this point, it is clear that the solutions for injection and withdrawal in any cycle of operation of an underground gas store are general. Suppose that l_i is the dimension of the gas store in the i-th cycle; σ_{i1} is the frontal value of the gas saturation after the i-th saturation; σ_{i2} is the frontal value of the water saturation after the i-th withdrawal; τ_{i2} is the duration of the i-th withdrawal. Then the determining relations take the form: for the n-th injection (n > 1)

$$\frac{\partial\sigma}{\partial\tau} + \frac{\partial f}{\partial\xi} = 0,\ \tau = n - 1 + \sum_{i=1}^{n-1}\tau_{0i},\ 0 \leqslant \xi \leqslant l_{n-1}:\ l_{n-1} - \xi = \tau_{0,n-1} f'(\sigma), \tag{6}$$

$$\xi \geqslant l_{n-1}:\ \sigma(\xi,\ 0) = 0,$$

$$\tau > n - 1 + \sum_{i=1}^{n-1}\tau_{0i},\ \sigma(0,\ \tau) = 1;$$

TABLE 1. Cycle Change in the Parameters of an Underground Gas Store in Approaching Steady Conditions of Operation

Cycle No., i	Rel. vol. of gas store, l_i/l_1	Rel. time of gas withdrawal, $\tau_{0i}=T_{0i}/T$	Frontal sat. after injection, σ_{i1}	Frontal sat. after withdrawal, σ_{i2}
1	1,00	0,54	0,29	0,18
2	1,52	0,71	0,27	0,18
3	1,88	0,81	0,26	0,19
4	2,14	0,88	0,25	0,19
5	2,32	0,91	0,24	0,19
6	2,46	0,94	0,24	0,19
7	2,55	0,96	0,23	0,20
8	2,63	0,97	0,23	0,20
9	2,68	0,97	0,23	0,20
10	2,72	0,98	0,23	0,20
15	2,83	0,99	0,22	0,20
∞	2,92	1,00	0,21	0,21

for the n-th withdrawal ($n \geq 1$)

$$\frac{\partial\sigma}{\partial\tau}+\frac{\partial f_1}{\partial\xi}=0,\quad \tau=n+\sum_{i=1}^{n-1}\tau_{0i},\quad 0\leqslant\xi\leqslant l_n:\ l_n-\xi=f_1'(\sigma), \tag{7}$$

$$\tau>n+\sum_{i=1}^{n-1}\tau_{0i},\quad \sigma(0,\tau)=1.$$

Since the structure of the solution is known, limiting relations for the gas-store parameters may be obtained. First of all, it is shown that the dimensions of the gas store do not increase without limit, but grow to some value l, which is the limit of the sequence $\{l_n\}$. The terms of this monotonically increasing sequence is given by the equation $l_n = f'(\sigma_{n1})$. The finiteness of the function $f'(\sigma)$ means that this sequence is finite and hence has the limit $l=\lim\limits_{n\to\infty} l_n$. Note, in passing, that the sequence of values of the frontal gas saturation σ_{n1} also has a limit. Since σ_{n1} belongs to the interval (σ_I; 1) in which $f'(\sigma)$ decreases monotonically, then the sequence $\{\sigma_{n1}\}$ also decreases monotonically. It follows from the relation $\sigma_{n1} > \sigma_I$ that $\lim\limits_{n\to\infty}\sigma_{n1}=\sigma_1\geqslant\sigma_I$.

It is now shown that the volume ratio of the gas pumped in and pumped out tends to unity. Note that, after the n-th injection, the gas volume $V_{3,n}$ in the plate is

$$V_{3,n}=\int_0^{l_n}\sigma d\xi=\int_1^{\sigma_{n1}}\sigma\frac{d\xi}{d\sigma}d\sigma=\int_1^{\sigma_{n1}}\sigma f''(\sigma)\,d\sigma$$

or

$$V_{3,n}=\sigma_{n1}f'(\sigma_{n1})-f(\sigma_{n1})+1. \tag{8}$$

The solution of the problem used here is in the form of a centered wave for which $d\xi/d\sigma = f''(\sigma)$. Since the same gas volume (unity) is injected into the bed in each cycle, the gas volume V_{0n} after the n-th withdrawal is determined as the difference of $V_{3,n+1}$ and unity, that is

$$V_{0n}=\sigma_{n+1,1}f'(\sigma_{n+1,1})-f(\sigma_{n+1,1}), \tag{9}$$

and the amount of gas withdrawn in the n-th cycle as the difference of $V_{3,n}$ and V_{0n}. Since $\lim\limits_{n\to\infty}\sigma_{n1}=\lim\limits_{n\to\infty}\sigma_{n+1,1}=\sigma_1$ and the functions f and f' are continuous, then $\lim\limits_{n\to\infty}(V_{3,n}-V_{0n})=1$ and the assertion has been proven. Hence, it follows, in particular, that the ratio of the durations of injection and withdrawal tends to unity, i.e., $\lim\limits_{n\to\infty}\tau_{0n}=1$.

Next, $\sigma_1=\lim\limits_{n\to\infty}\sigma_{n1}$ is calculated, and it is shown to be equal to σ_I, the abscissa of the point of inflection of the Buckley–Leverette function. In this case, analogously to the derivation of Eq. (8), an expression is obtained for the gas volume V_{0n} remaining in the bed after the n-th withdrawal

$$V_{0n}=\int_0^{l_n}\sigma d\xi=\tau_{0n}\,[f'(1-\sigma_{n2})(1-\sigma_{n2})-f(1-\sigma_{n2})]. \tag{10}$$

Since V_{0n} and τ_{0n} have limits, and $\tau_{0n} \to 1$, then there is a limit σ_2 of the sequence $1 - \sigma_{n2}$. Comparing Eqs. (9) and (10), it is found that

$$\sigma_1 f'(\sigma_1) - f(\sigma_1) = \sigma_2 f'(\sigma_2) - f(\sigma_2). \tag{11}$$

Since $1 - \sigma_{n2} < \sigma_I$, then $\sigma_2 \leq \sigma_I$; note that $\sigma_1 \geq \sigma_I$.

It may be established from Eq. (11) that $\sigma_1 = \sigma_2 = \sigma_I$. In fact, this equation has a simple geometric interpretation. The expression $\sigma f'(\sigma) - f(\sigma)$ determines the length of the intersect [OA] on the ordinate formed by the continuation of the tangent to the Buckley curve drawn at a certain point. Equation (11) requires that the points $B_1(\sigma_1)$ and $B_2(\sigma_2)$ (Fig. 4) lie on this curve in such a way that the tangents drawn at these points form the same intersect on the ordinate. It may readily be noted here that the tangent (AB_1) to the upper part of the Buckley curve always passes higher than the tangent (AB_2) to the lower part (when $\sigma_1 = \sigma_2$, they coincide, of course). Assume that $\sigma_1 \neq \sigma_2$, i.e., $\sigma_2 < \sigma_1$. Since, with alternating pumping, a stable transition from the saturation value σ_2 before the discontinuity to the value of unity after the discontinuity is achieved by a jump along the Buckley curve from point B_2 to a point of section [B_1C] and subsequent continuous motion along the centered wave, inability to draw a tangent from point B_2 to the given segment of the Buckley curve means that the assumption made has led to a contradiction. Hence, $\sigma_1 = \sigma_2$. Taking into account that $\sigma_1 \geq \sigma_I$ and $\sigma_2 \leq \sigma_I$, it follows that $\sigma_1 = \sigma_2 = \sigma_I$.

Thus, the steady operation of an underground gas store is characterized by such parameters of the Buckley curve as σ_I, $f(\sigma_I)$, and $f'(\sigma_I)$. At the same time, the dynamics of the transition of the gas store to steady conditions of operation is determined by the behavior of this curve over the whole range of variation of σ.

The most significant parameters of steady operation of the gas store are as follows.

1. The limiting dimension of the gas store

$$l = f'(\sigma_I). \tag{12}$$

This result is especially important for explorative work to determine the possibilities of a particular bed for use as an underground gas store, and also in calculations of the amount of gas which may be stored there.

2. The volume of the trapped – i.e., inaccessible for removal – gas actually consumed in creating the gas store

$$V_0 = \sigma_I f'(\sigma_I) - f(\sigma_I). \tag{13}$$

This value is represented graphically by the intersect [OA] (Fig. 4).

3. The extraction coefficient φ (the volume ratio of the gas extracted and the gas stored)

$$\varphi = \frac{1}{1+V_0} = \frac{1}{1+\sigma_I f'(\sigma_I) - f(\sigma_I)}. \tag{14}$$

This coefficient is the efficiency index of the operation of the underground gas store.

4. The mean gas saturation after injection $\bar{\sigma}_1$ and withdrawal $\bar{\sigma}_2$

$$\bar{\sigma}_1 = \frac{\sigma_I f'(\sigma_I) - f(\sigma_I) + 1}{f'(\sigma_I)}; \quad \bar{\sigma}_2 = \frac{\sigma_I f'(\sigma_I) - f(\sigma_I)}{f'(\sigma_I)}. \tag{15}$$

The dynamics of gas-store departure to steady conditions of operation is determined, as already noted, by the specific form of the Buckley function. This process is characterized by a sequence of times τ_{0i} and hence the volumes of gas withdrawn from cycle to cycle. These quantities have been calculated on a computer by integration of ordinary differential Eq. (4). The experimental curve of [7] was used as the Buckley function. The limiting dimension l of the gas store in this case is 8.72 ($l/l_1 = 2.92$, i.e., the gas store may be increased by no more than a factor of 2.92 in comparison with the first injection); the extraction coefficient $\varphi = 0.41$; the mean gas saturations $\bar{\sigma}_1 = 0.28$, $\bar{\sigma}_2 = 0.17$. The sequence of τ_{0i} for a few cycles is given in Table 1, from which it is evident that the difference between the withdrawal and the injection time becomes less than 3% in approximately the eighth cycle of gas-store operation.

NOTATION

σ, saturation of one of the cofiltering fluids (gas or water); m, porosity; w, total specific filtrational flow rate; $f(\sigma)$, $f_1(\sigma)$, Buckley–Leverette function; $k_1(\sigma)$, $k_2(\sigma)$, relative phase permeabilities; μ_0, viscosity ratio of phases; σ_I, abscissa of the point of inflection of the Buckley function; V, gas volume injected into the bed; T, duration of pumping; T_{0i}, duration of i-th gas withdrawal; t, time; x, spatial variable; τ, dimensionless time; ξ, dimensionless spatial coordinate.

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TWO TYPES OF HEAT TRANSFER IN MEDIA WITH THERMAL MEMORY

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UDC 536.24

It is shown that media with thermal memory can be grouped into two classes, based on different types of heat transfer. In media of the first class, the heat propagation velocity is infinite, while in the second class, it is finite. This difference is responsible for the peculiarities of the solutions of the heat-conduction problem in the two classes.

Currently in the study of heat- and mass-transfer processes under extreme conditions (low or very high temperatures), the mathematical formulation of heat conduction and mass exchange is used including differential memory of the medium [1-4, 7-9]. A linerarized heat-conduction equation of this kind was first obtained in [8]; it describes heat transfer with a finite heat propagation velocity [8, 9]. In the derivation of a similar heat-conduction equation in [2], a different, more general form of the linearized integral heat-transfer relation was used, which includes the instantaneous values $\lambda(0)$ and $c(0)$ of the relaxation functions for the heat flux and the internal energy. Then media with transient thermal memory can naturally be divided into two classes: those with the instantaneous value $\lambda(0) > 0$ (Fourier media) and those with $\lambda(0) = 0$ (Maxwellian media). It was also shown in [2] that the Nunziato heat-conduction equation with $\lambda(0) = 0$ can be reduced to the Pipkin–Curtin equation [8] and hence in this type of medium, heat propagates with a finite velocity. It is shown below that in a Fourier medium, heat is transferred with an infinite velocity. Using the method of solving the heat-conduction problem for the Nunziato equation worked out in [4], we describe the heat-conduction behavior for small values of the time in both types of media. The results are applied to the distribution function of an instantaneous point source and this allows one to deduce the type of heat propagation and also the qualitative features of the solution for each type of medium.

We consider the integrodifferential heat-conduction equation for the function $u(t, M) = T(t, M) - T(0, M)$ describing the linearized transfer process with transient thermal memory as formulated by Nunziato [2]:

$$\frac{c_1(0)}{a_0}\frac{\partial u}{\partial t} - \lambda_1(0)\Delta u + \int_0^\infty \left[\frac{1}{a_0}\frac{dc_1(\tau)}{d\tau}\frac{\partial u(t-\tau, M)}{\partial t} - \frac{d\lambda_1(\tau)}{d\tau}\Delta u(t-\tau, M)\right]d\tau = \frac{b(t, M)}{\lambda_0}, \quad (1)$$

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